

Section 8-5: Matrices and Determinants

By the end of this lesson, you should be able to answer:

- How do you find the determinant of a 2X2 matrix?
- How do you solve systems of equations using determinants?

Where you might see this in the real world:

- Sports, construction, fitness

Define the following terms:

1. Square matrix
2. Determinant
3. Cramer's rule

A matrix is a rectangular arrangement of elements, listed as $m \times n$, where m is the number of rows and n is the number of columns. A square matrix is a matrix that has the same number of rows and columns. Here are some examples of matrices:

$$\begin{bmatrix} -2 & 2 & 3 \\ 9 & 6 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 5 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 & 0 \end{bmatrix}$$

In this lesson we will focus on square matrices, namely 2X2 matrices. Every square matrix has a determinant. When we are looking for the determinant, we will have vertical bars instead of the square brackets.

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Here, the a , b , c , and d are just variables that represent the position of the elements in the matrix. Here's how we use the determinant.

Example 1: Find the determinant of $\begin{bmatrix} 0 & 4 \\ 6 & 7 \end{bmatrix}$.

This is an idea that comes in handy when solving systems of equations. Gabriel Cramer found a rule that uses the determinants of matrices that are pulled from systems can be used to find the solutions by using the following rules:

1. Make sure both equations are written in standard form $Ax + By = C$
2. Write a 2×2 determinant matrix with the two equations. Your x coefficients go in the first column and the y coefficients go in the second column. Name this determinant A .
3. Write another determinant matrix, but replace the x -column with the two constants of the equations. Name this determinant A_x .
4. Write a third determinant matrix, this time replacing the y -column with the two constants. Name this determinant A_y .
5. To find the x -coordinate of the solution, divide A_x by A , then divide A_y by A to find the y -coordinate.
6. Finally, check your answer and rewrite as an ordered pair.

Example 2: Solve the system of equations using Cramer's rule (matrices).

$$\begin{cases} 3x - 7y = -6 \\ x + 2y = 11 \end{cases}$$

Problem Set:

"I'm a great believer in luck, and I find the harder I work the more I have of it." - Thomas Jefferson